Understanding Famous Structures Through Simple Graphical Analyses

EDWARD ALLEN
Yale University

with contributions by
WACLAW ZALEWSKI
Massachusetts Institute of Technology

The great structures of the modem era—works of such masters as Eiffel, Maillart, Nervi, Ammann, Arup, and Calatrava—are often held up as examples that represent a level of design achievement to which students of architecture should aspire in their own structures. Yet we give students little help in understanding how these works were conceived or in knowing how they might go about emulating them. Our teaching of structures tends to concentrate narrowly on calculations of stresses and deflections in beams and columns, seldom venturing into the realm of curvilinear, longspan forms that these structures represent, or into the process of synthesizing structural form. A glance at any contemporary engineering text on arches, cables, or shells leads to the immediate but erroneous conclusion that one can only attempt to design such devices by employing forms of mathematics that are beyond the reach of most students of architecture. The published writings of the master designers themselves offer few clues concerning the origins of their graceful works and tend to perpetuate a myth that only a few select people of great genius are capable of creating such elegant forms. As a result, attempts by students to design structures other than rectilinear frames tend to be hesitant, clumsy, and poorly informed.

There is a way of demystifying the great structures, giving students both a vastly increased understanding of how they were designed and a confident ability to design structures that are similar in principle and equally rational in concept. It involves the use of simple, astonishingly powerful graphical techniques that were used by the master designers themselves to create many of their most admired structures. These techniques are referred to collectively as graphic statics.

Graphic statics were perfected in the middle of the 19th century and were widely used by architects and engineers until well into our own century.[1] In recent decades they have been largely abandoned in favor of numerical techniques, but vestiges remain in the familiar guises of shear and moment diagrams and the graphical analysis of the forces in trusses.

The graphical analysis of a truss is easily understood, and is shown sequentially for a typical truss in Fig. 1. Working with drafting instruments, we first draw a free-body diagram of the entire truss accurately to scale; this is called the space diagram (Fig. 1a). A labeling system known as Bow's notation is used to keep track of members and forces—letters are applied to the spaces between the external loads, and numbers to the spaces bounded by members of the truss. Next, alongside the space diagram, we plot the external loads sequentially on a load line to any convenient scale of length to force, labeling the ends of the line segments with lower-case letters that correspond to the capital letters that lie on either side of the corresponding force on the space diagram (Fig. 1b). Then pairs of intersecting lines are drawn off appropriate points on the load line, accurately parallel to pairs of intersecting truss members on the space diagram, to create linked equilibrium polygons whose sides are scaled to find the magnitudes of the forces in the truss members without doing any numerical computations (Fig. 1c-g). A clockwise convention is applied to this diagram, using Bow's notation, to determine whether each member of the truss is in tension or compression. The completed diagram of equilibrium polygons is traditionally called either a Maxwell diagram or a Cremona plan; in this paper it is called a Maxwell-Cremona diagram. Because the analysis in Fig. 1 includes only gravity loads, the load line is vertical. Inclined loads such as wind loads on a sloping roof surface may be analyzed in similar fashion. In such a case, the load line will include inclined segments. A valuable feature of graphical truss analysis is that it is self-checking: If the last line segment on the Maxwell-Cremona diagram does not meet the appropriate points accurately, an error has been made.[2]

This graphical process gives us a way of examining and understanding trussed structures that is often more revealing and convenient than numerical methods. In Fig. 2, we use it to discover Eiffel's assumptions about wind load distributions for his 300 meter tower in Paris. We do this by treating the entire tower in simplified fashion as a funicular cantilevered truss of four panels that is subjected to concentrated horizontal loads of unknown magnitudes at the panel points. The load line, accordingly, is horizontal, and we assign to it an arbitrary length that represents 100% of the wind force on

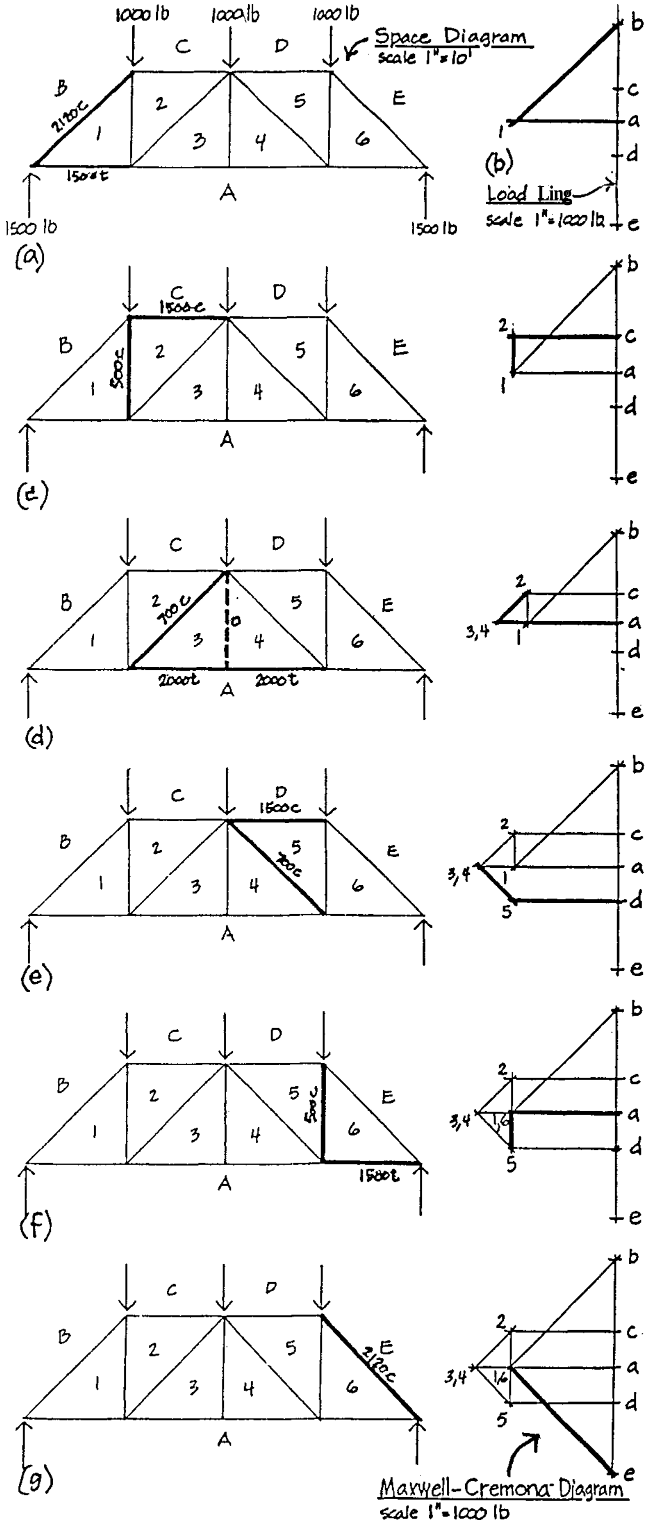

Fig. 1

the tower. The panels in Eiffel's design lack diagonals. To carry out a standard graphical analysis, we must insert fictitious diagonals and assign them forces of zero. This having been done, the analysis proceeds smoothly, yielding the proportional magnitudes of the wind forces at the panel points as well as the member forces that these induce. Although Eiffel assumed unit wind pressures to be higher at the top of the tower than at the bottom, the vastly greater exposed area of the lower portions of the tower accounts for the much larger total forces that act there. The Eiffel Tower is a particularly apt choice for this type of examination, because it was designed using graphical methods by Eiffel's assistant, Maurice Koechlin.[3]

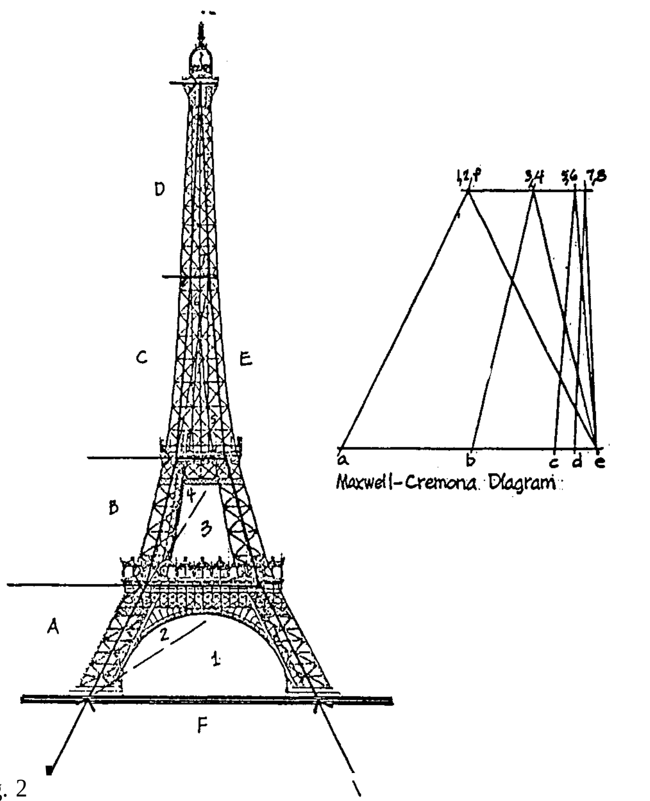

Fig. 2

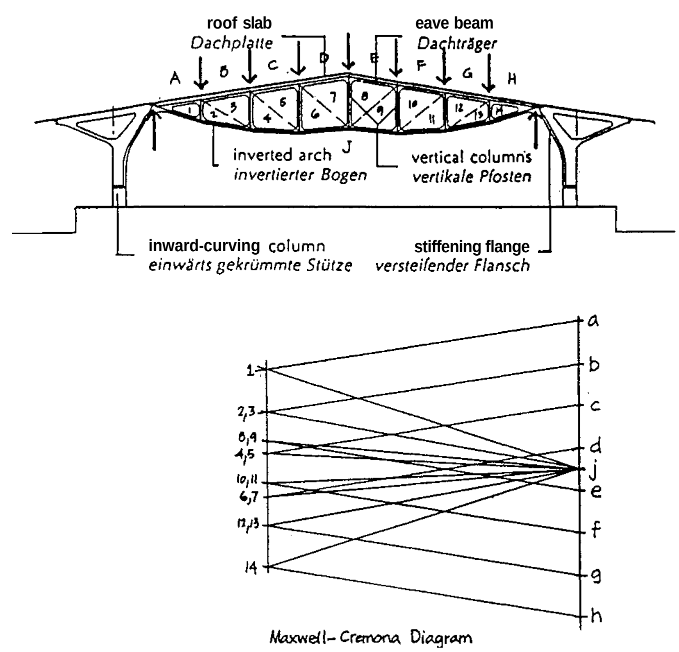

Fig. 1

Fig. **3** shows a graphical analysis of the forces created by a uniform distribution of gravity loads in Robert Maillart's trusses for the Magazzini Generali roof in Chiasso. As we did with the Eiffel Tower, we must insert fictitious diagonals in the rectangular panels and assign them zero force. The Maxwell-Cremona diagram shows the ingenuity of Maillart's seemingly inexplicable truss form with startling clarity: The forces in the sloping upper chords are constant throughout, thus simplifying the construction of the roof and eliminating the need for diagonal members. The question may be asked, how did Maillart synthesize a truss form that has these unique properties? It is easy to demonstrate how he might have done so by constructing the Maxwell-Cremona diagram first and using it to determine the construction of the space diagram, drawing it in such a way that the upper chord segments all have equal force while the diagonals have none. The fan of line segments that converge on point *j* then give the inclinations and locations of the lower chord members, allowing the space diagram to be completed.[4]

Closely related to the graphical method for analyzing trusses is a simple graphical construction for plotting the curve of a hanging cable and measuring its internal forces. This opens vast new horizons to students, for by this means, without using numerical mathematics, they can easily find form and forces for hanging structures and arched structures, either those of master designers or ones of their own design. Fig. 4 is an analysis of the effect of a uniformly distributed load on the hanging roof at Dulles Airport that was designed by Eero Saarinen with structural engineering by Ammann and Whitney. Working from a published cross section, the roof is divided into convenient segments, loads are estimated, a load line is drawn, and the force diagram is constructed with line segments parallel to each of the segments of the roof. The force in any segment of the cable may be scaled from this diagram and an appropriate diameter of cable selected from a catalog.

If we invert the form of a hanging cable with a given distribution of loads, we have an ideal shape for an arch that supports the same loading. Thus we may use a similar type of diagram to analyze Pier Luigi Nervi's arched roof for the Turin Exhibition Hall (Fig. 5). The plotted funicular curve for a uniform gravity loading is a perfect fit to the centerline of Nervi's vault. Having taken cross-sectional area measurements from a detail of the vault construction, an approximate maximum stress in the concrete of the vault (about 280 psi) is easily calculated. A useful extension of this exercise is to ask students to plot funicular curves for unbalanced snow loadings and wind loadings on this roof, and to discuss these in relation to the corrugated form that Nervi gave to its longitudinal section.

Fig. 6 shows an analysis of balanced and unbalanced loadings as applied to Robert Maillart's much-admired Salginatobel Bridge. The righthand force diagram is used to find the line of pressure in the bridge for dead load only, and

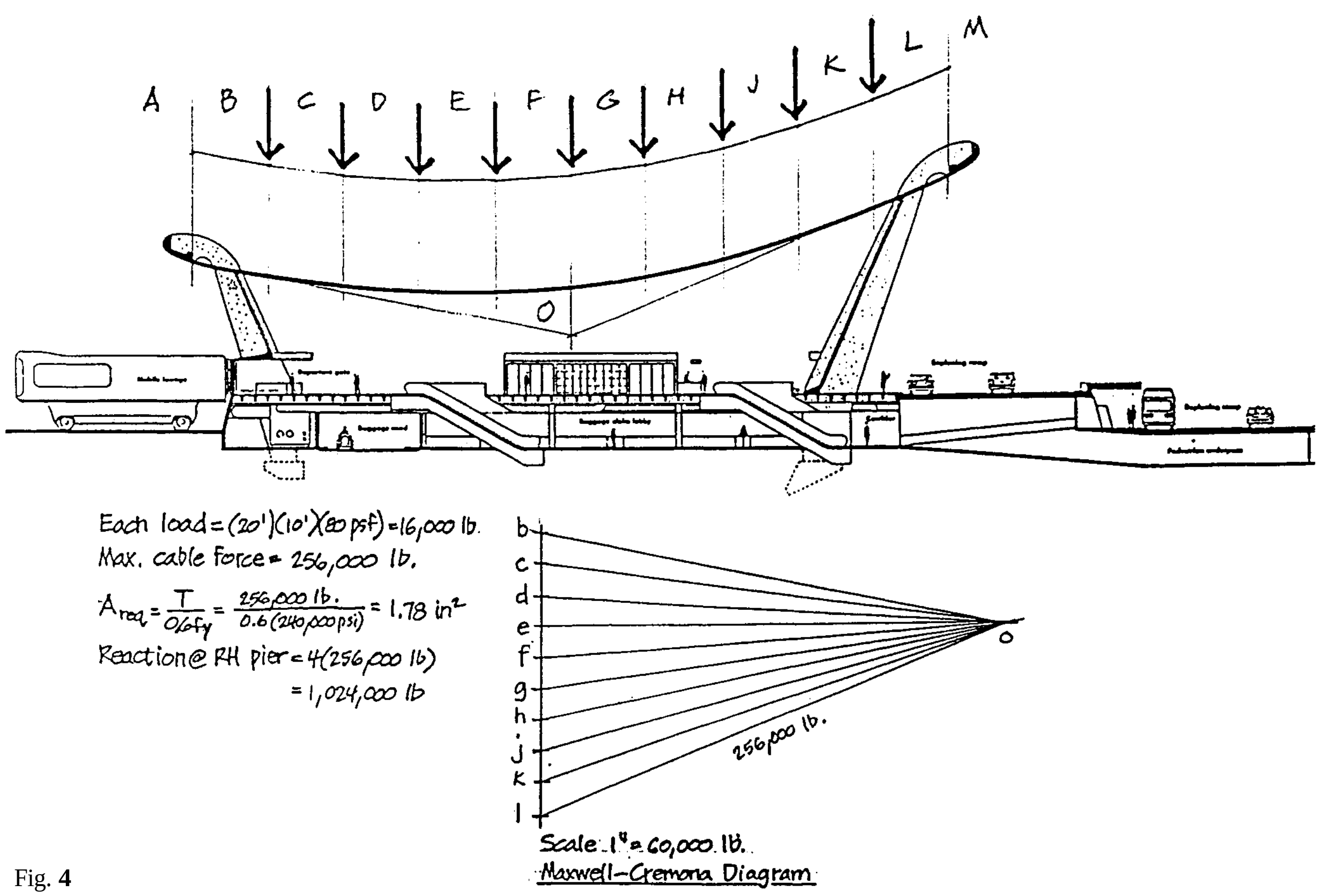

Fig. **4**

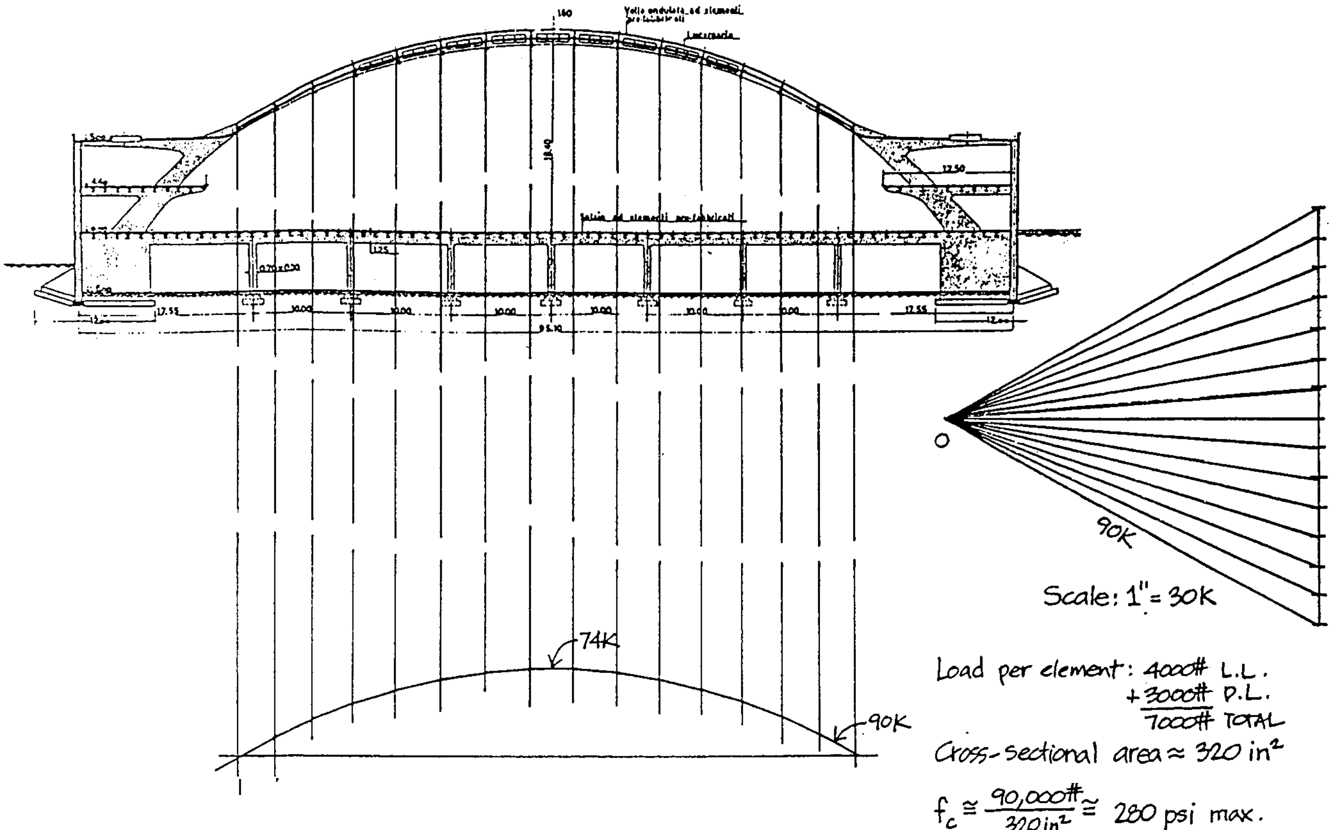

Fig. 5

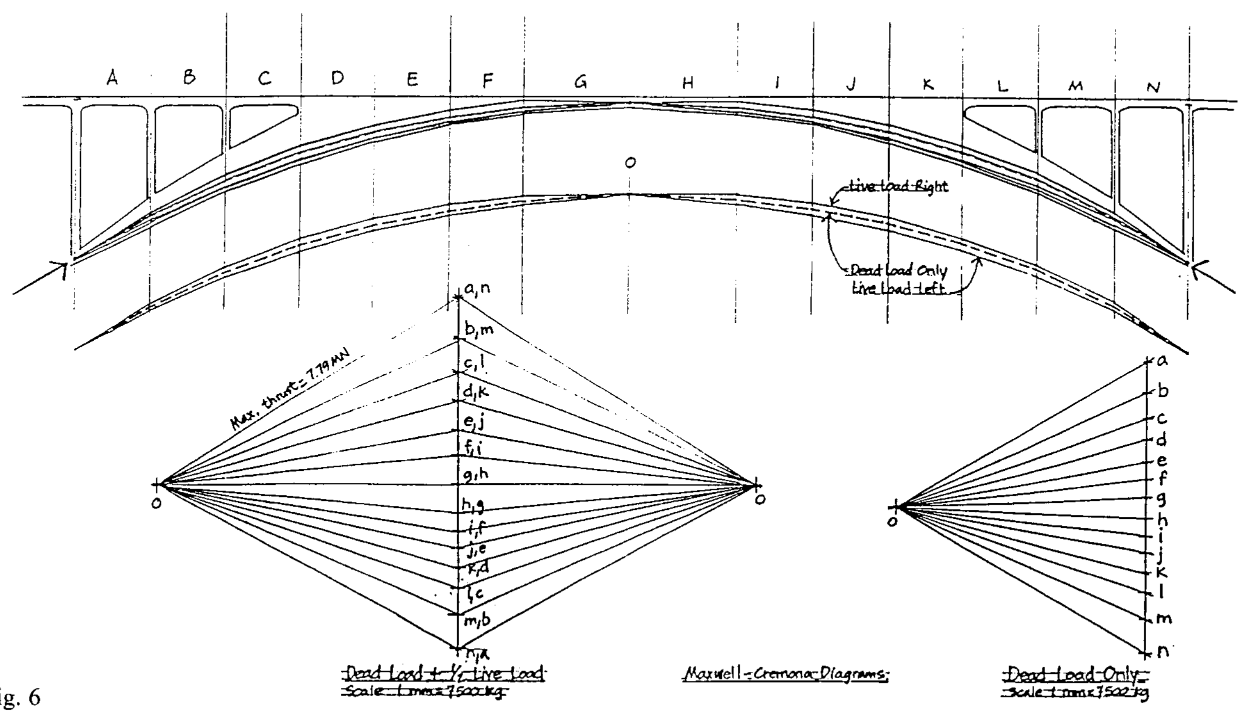

Fig. 6

is based on careful estimates of component weights from dimensions given on Maillart's own drawings. The lefthand diagram serves to plot lines of pressure for unbalanced loadings of dead load plus live load on either half of the bridge. The locations of the poles for the diagrams, points o, are manipulated in such a way that all three lines of pressure pass through the three hinges of the arch.[5] For clarity, the lines of pressure are drawn twice on this diagram, once in isolation below the elevation of the bridge, and again superimposed upon it. The two slender boomerang shapes thus visualized make it easy to understand the logic of the well-known profile of the bridge, especially if one takes the time to examine the cross-sections of the box arch and find the limits of their kerns in relation to the locations of the lines of pressure. Drawings viewed by the author in the Maillart Archive at the ETH in Zurich show that Maillart designed the bridge by the same graphical methods used in this analysis.

Graphical truss analysis can be applied directly to certain types of structures that are not usually thought of as being trusses, such as the cable-stayed roof of the Patscenter building in Princeton, New Jersey, by Richard Rogers and Ove Arup. The simple investigation shown in Fig. 7 explains

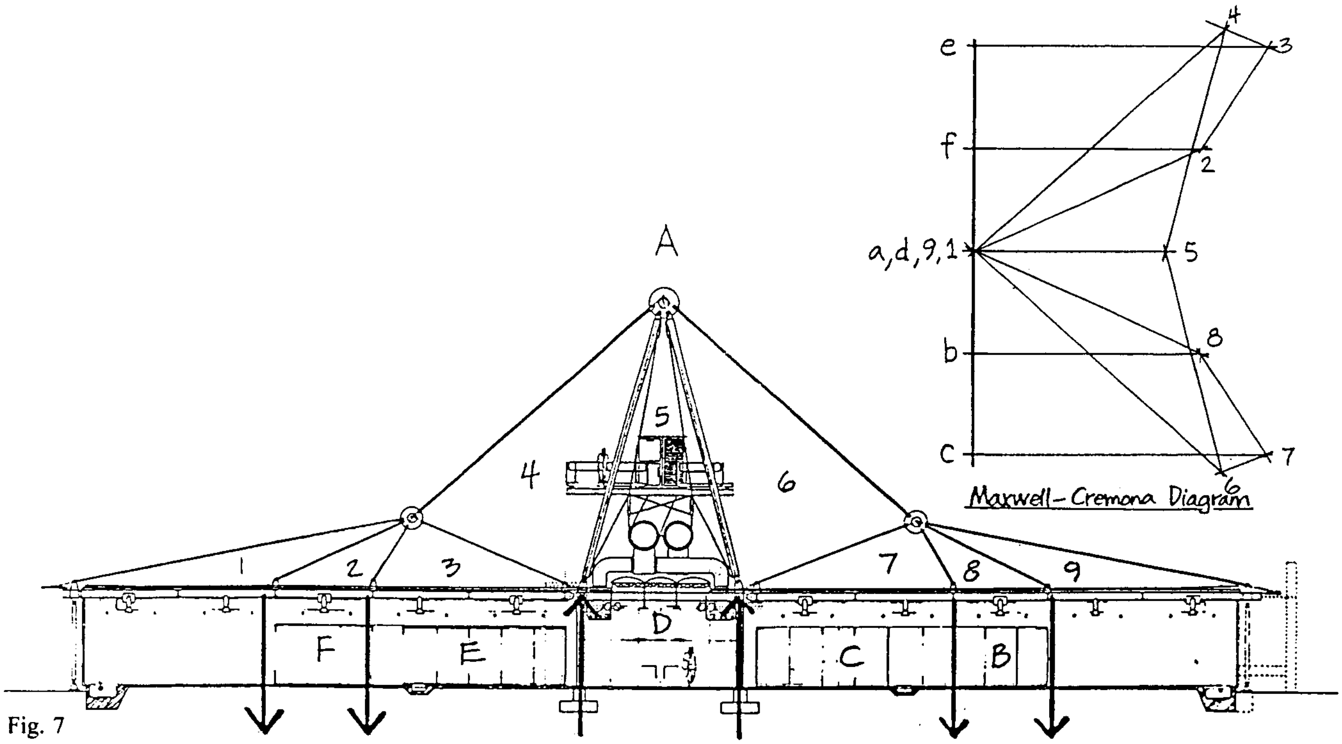

Fig. 7

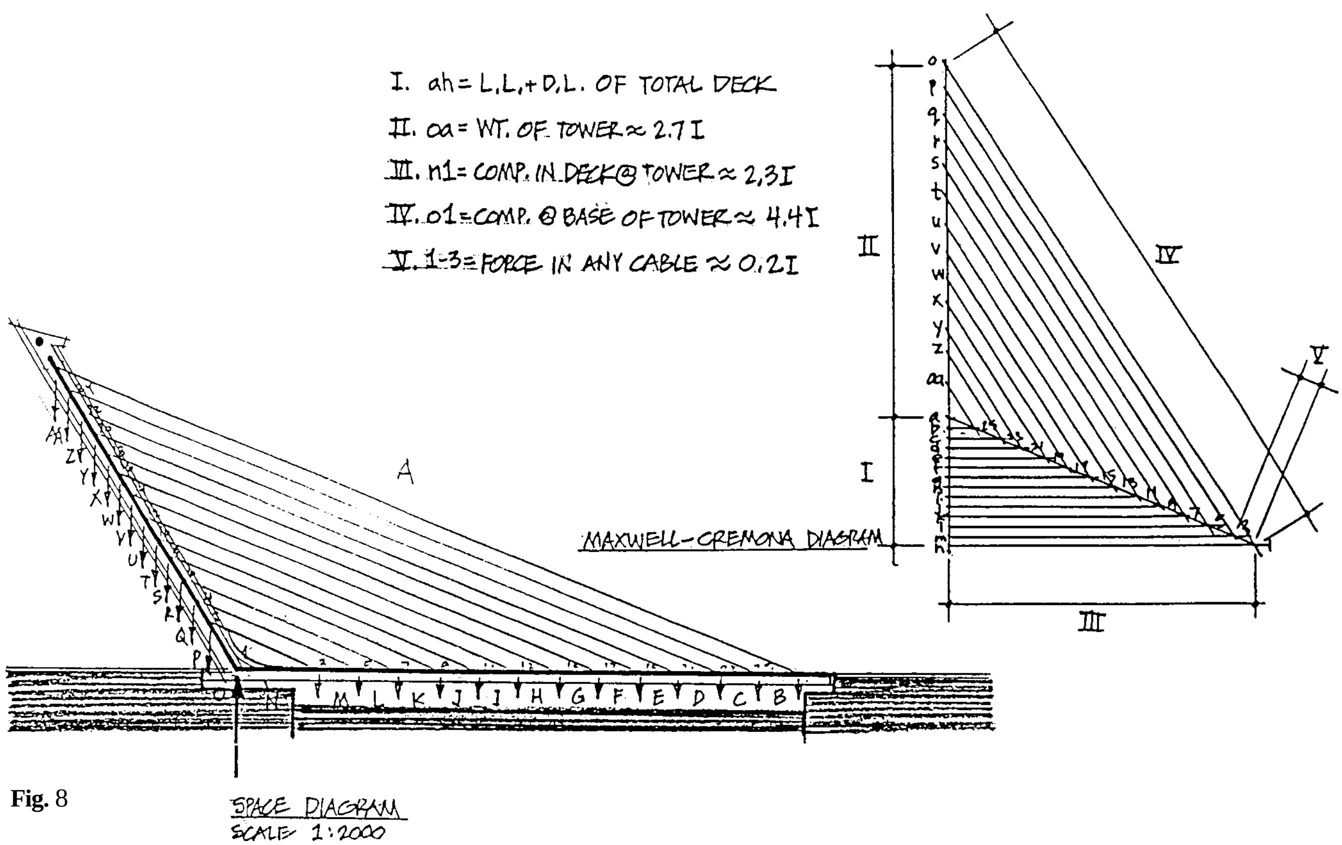

Fig. 8

the action of the structure in a way that students can understand and, given some assumptions about loadings, it quantifies the forces in all the members of the roof, including the axial compressions introduced into the roofbeams by the inclined stays. It discloses that two of the tensile members carry no force under a uniform roof loading, and two others carry only minimal tension, but that all four would play crucial roles in resisting unbalanced snow or wind loads. This discovery can lead to a useful discussion of various ways to deal with nonuniform loadings in cable-stayed roofs. An analysis such as this opens to students the possibility of designing cable-stayed structures for their own studio projects. Learning the member forces in their designs through simple graphical analysis, they can easily size the tension rods, cables, and columns, then proceed to development of the connection details.

A single Maxwell-Cremona diagram makes it possible to summarize all the primary features of the structural performance of Santiago Calatrava's spectacular Alamillo Bridge (Fig, 8). We represent the loads of the deck sections by the increments that make up segment *an* of the load line. Given the pylon's published inclination of 59 degrees, its required weight, represented by segment *oa* of the load line, is developed as a function of the deck load in the course of constructing this diagram without doing any numerical calculations. The forces in the cables and in each segment of the deck and pylon may also be scaled from this diagram. Using this diagram, it is interesting to speculate as to what load Calatrava assumed for the deck as a basis for determining the weight of the pylon—was it the dead load only, dead load plus full live load, or something between, counting on the stiffness of the pylon to balance other loading conditions? Students can be asked to experiment with the consequences of changing the inclination of the pylon, increasing its height, or supporting the deck from towers at both ends, all by making simple modifications to this diagram.

A couple of generations ago, a student of architecture in England, obviously very facile with graphic statics, did as his thesis project a thorough graphical analysis of the structure of a gothic cathedral.[6] Using the same techniques that we have applied here to modern structures, he examined not only the curves of the arches and vaults and the lines of thrust in the flying buttresses, but even such details as the wind load bending stresses in the slender limestone window mullions. In the course of his work he unlocked many a secret of the medieval master builder. He discovered the crucial structural role of the heavy stone boss at the crown of each pointed vault, and of the hidden masonry fill at its springing. His is a particularly rich example of how simple graphical explorations can help us and our students to unlock the secrets of many famous structures and their designers.

As a byproduct of even greater value, these graphical methods can furnish students with powerful synthetical tools for designing their own longspan structures, for they are suited perfectly to studio work. The author has provided students with these techniques in both classroom and design studio for nearly twenty years. Samples of student work were shown during the presentation of this paper, but space is too limited in this publication to include these projects.

Students find graphical techniques for structural design fascinating and easy to learn. Starting from scratch, it takes no more than an hour of class time and one homework exercise for students to learn the graphical analysis of trusses. Another class period suffices to extend the technique to suspended and arched structures, after which students have at their command the fundamental means by which the many of the great structures of the last 150 years were given form. Other teachers may find graphical methods useful both as design tools for student use, and, as illustrated here, as a means for analyzing and understanding famous structures.'

NOTES

1 For a brief history of graphic statics, see Hans Straub, *A History of Civil Engineering,* Cambridge MA, MIT Press, 1964, pp. 197-202.

2 For a more extended description of graphical methods for truss analysis, see James Ambrose, *Design of Building Trusses,* New York, John Wiley & Sons, Inc., 1994, pp. 331-377. These pages largely duplicate verbatim the excellent treatment of the subject by the late Harry Parker.

3 Koechlin's use of graphic statics in the design of the Eiffel Tower is documented in a book, *La Tour de Trois Cent Metres,* that was published by the Eiffel office through Imprimerie Mercier in 1900.

4 For a more detailed and extended presentation of graphical derivations of ideal truss forms, see Edward Allen, "Finding Efficient Forms for Trusses," in Ambrose, *op. cit.*, pp. 378-395.

5 A funicular curve for any set of loads may be constructed through any three points by utilizing a graphical construction shown in Paul Andersen and Gene M. Nordby, *Introduction to Structural Mechanics,* The Ronald Press Co., New York, 1960, pp. 61, 62. Another source of this method is William S. Wolfe, *Graphical Analysis*, New York, McGraw-Hill, 1921, pp. 37-39.

6 Gerhard Rosenberg, "The Functional Aspect of the Gothic Style," *Journal of the Royal Institute of British Architects,* 18 January 1936, pp. 273-290.

7 The best general references on graphic statics are Wolfe, Andersen and Nordby, and Ambrose/Parker/Allen, refs. 2 and 5 above.